Applying a Throughput Stability-Condition Model to a Picking Machine

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Abstract

Picking machines are an example of a stock-to-picker fulfillment technology that consists of two or more pick stations and a common storage system. A conveyor system decouples the pick stations from the storage area by transporting the needed totes between the storage system and the picking stations. We develop an analytical stability-condition model based on queuing theory to determine if a picking machine’s closed-loop conveyor will be able to meet a throughput requirement. Through simulation, we show that our stability-condition model accurately predicts whether a closed-loop conveyor will meet throughput even when model assumptions regarding Poisson arrivals and infinite buffer capacities are violated.

Keywords
Facility Logistics, Order-Fulfillment Technology, Closed-Loop Conveyor Systems, Queuing Theory

1. Introduction

A picking machine, also known as a remote order picking system [1] or an automated storage and order fulfillment system [2], is a stock-to-picker order-fulfillment technology that consists of numerous pick stations that access a common inventory storage area, which is usually a carousel system or a mini-load automated storage and retrieval system (AS/RS) where stock keeping units (SKUs) are stored in totes. A picking machine serviced via a carousel system with robotic pickers is depicted in Figure 1. An integrated conveyor system transports the needed totes to the picking stations when needed; therefore, even though a picking machine has numerous pick stations, they all utilize the same storage system. The required orders’ shipping containers are transported via a conveyor to a pick station. At the same time, the requested SKUs’ totes are retrieved automatically from the storage system and are also sent to the pick station. At the pick station, pick lights indicate to the operator both the position and the quantity of the items to be picked and put lights direct the operator to the position of the container to which the items should be transferred.

Figure 1: A Picking Machine Serviced by a Carousel System (Photo Courtesy of SSI Schaefer)
As shown in Figure 2, a picking machine can be thought of as a network of queues with three subsystems: a storage area, pick stations, and a closed-loop conveyor. Totes arrive to the subsystems and may need to wait in a queue before being utilized. After being served, the totes enter the queue of another subsystem (i.e., totes can be transported from storage to the conveyor, from the conveyor to a pick station, from a pick station to the conveyor, and from the conveyor to storage). Therefore, an upper bound on the throughput of a picking machine is the minimum of the three subsystem’s throughputs.

Figure 2: A Picking Machine as a Closed-Loop Queuing Model

Because a closed-loop conveyor integrates the storage and picking stations in a picking machine, a key to modeling a picking machine is to model the closed-loop conveyor. Therefore, we focus on determining if the closed-loop conveyor will be able to meet a throughput requirement, and in the next section review closed-loop conveyor literature.

2. Literature Review
Conveyor systems are used to transport units from one area to another and are prevalent in many service, manufacturing, and distribution facilities. A review on modeling conveyor systems and theory is provided in [3]; we focus our attention on recent research that uses queuing theory to model closed-loop conveyor systems.

Bozer and Hsieh [4] analyze the performance of discrete-space, fixed-window, closed-loop conveyors with multiple loading and unloading stations. They determine if the conveyor loop meets a throughput requirement by developing a stability condition that focuses on conveyor loading stations. The conveyor is divided into segments and each segment is modeled as an independent M/G/1 queuing system with exceptional first service. The conveyor acts as the server and arrivals to input stations are assumed to follow a Poisson process. A type-1 load is a load that finds an empty input queue upon arrival to the loading station. A type-2 load is a load that finds a non-empty input queue upon arrival to the loading station. Because type-1 loads initiate a busy period and have “exceptional” service, results from Welch’s M/G/1 queues with exceptional first service [5] are applied. They define a stability factor for each segment and the closed-loop conveyor system is stable if and only if every segment is stable.

Bozer and Hsieh find that steady-state conveyor stability depends only on the arrival rate to each loading station, the flow rate in each conveyor segment, and the conveyor velocity. The number of windows on the conveyor does not impact steady-state conveyor stability. The stability condition correctly predicted the outcome for each loading station in all instances tested by Bozer and Hsieh with Poisson arrivals and infinite buffer capacities.

Hsieh and Bozer [6] extend their earlier work by allowing for finite buffer capacities at unloading stations. Loads that encounter a full queue are blocked and recirculate around the conveyor loop and try again. Loads are assumed to arrive to loading stations following a Poisson process and input stations assumed to have infinite capacity. Hsieh and Bozer use conditional probabilities to determine the probability of blocking at an unloading station by estimating the number of attempts a load makes before the load is successfully unloaded at a particular station. Using this probability, they estimate the expected load overflow on the conveyor due to blocked loads, which can then be combined with results from [4] to develop a stability condition for a closed-loop conveyor with finite capacity at unloading stations.

Bozer and Hsieh [7] approximate the expected waiting time for loads arriving at loading stations in stable, discrete-
space, fixed-window, closed-loop conveyor systems with infinite capacity at input and output queues. Although the time between successive open windows are statistically independent, as shown in [4], the status of adjacent windows is found to be correlated and has a significant impact on the expected waiting times.

Hsieh et al. [8] evaluate a system that integrates a conveyor with a single rotary rack system. In this system loads arrive to a loading station one at a time according to an independent Poisson process, enter a queue, and are loaded onto a conveyor destined for the rotary rack. They develop expected single-command cycle times for the system that incorporate the waiting time at the conveyor loading station and at the rotary rack. The system studied in [8] is a simplified version of the system we study as the system consists of a single loading and unloading station, whereas our system has multiple loading and unloading stations around a closed-loop conveyor.

Maggio et al. [9] study the problem of a closed-loop, three-machine production system with unreliable machines, finite buffers, and a fixed population. They present an approximate analytical method for evaluating the average values of throughput and buffer levels using a decomposition method that accounts for the correlation among the number of parts in the buffers. Gerswin and Werner [10] present a similar approach that addresses the increased complexity of larger systems with more than three machines.

Lazaro and Perez [11] analyze a network of closed-loop conveyors decoupled by intermediate buffers in an automobile assembly line. The population of items in the system is not fixed and machines can work at different cycle times. They analyze the relationship between the cycle times of the machines and the number of pallets in their upstream buffers. In a transitory regime, the number of pallets in any intermediate buffer can be lower than the minimum required to guarantee a working station. An exponential decay function characterizes the number of pallets in upstream buffers for a transitory regime. On the other hand, the cycle times remain constant in a stationary working regime. This work is extended in [12] to include starving and blocking in the system. They find that in a transitory regime (e.g., the number of pallets in any intermediate buffer can be lower than the minimum required to guarantee a working station) depends on the number of pallets in their upstream buffers according to exponential decay functions, but the cycle times remain constant in the stationary working regime.

Smith [13] analyzes a system of workstations that are interconnected with a material handling system. He uses \( M/G/1/K \) queuing models to measure the material processing delays and \( M/G/c/c \) queuing models for the material handling flows in the system. A conveyor network is modeled using a Steiner minimal tree with the objective to minimize the amount of conveyor length used.

3. Stability-Condition Model
As previously stated, a key to modeling a picking machine is to model the closed-loop conveyor as it integrates the storage and picking stations (PS). We extend Bozer and Hsieh’s stability functions for a closed-loop conveyor [4] to develop an analytical model to determine if a picking machine’s closed-loop conveyor is stable in steady state for a given throughput. Our stability-condition model offers an alternative to simulation, which can be expensive and time consuming to construct, for analyzing the complex nature of picking machine technology. First, the following notation is defined and visually displayed in Figure 2.

In our analysis we denote a queue that is formed by totes waiting to be placed on the conveyor as an on-queue and a queue that forms by totes being ejected off of the conveyor as an off-queue.

3.1 Notation

Sets:
- \( C \) Carousel Systems; indexed on \( c; c = 1, 2, \ldots, |C| \)
- \( O \) Pick Stations; indexed on \( o; o = 1, 2, \ldots, |O| \)
- \( Q \) Conveyor on-queues; indexed on \( q; q \in C \cup O \)
- \( K \) Conveyor off-queues; indexed on \( k; k \in C \cup O \)

Parameters:
- \( \mu_o \) the expected service rate at pick station \( o \)
- \( V \) the speed of the conveyor in the number of windows moved per time unit
- \( \chi_{q,k}^i \) 1 if a tote from on-queue \( q \) to off-queue \( k \) travels by queue \( i; 0 \) otherwise, \( (i \in Q \cup R) \)
- \( E[CT_c] \) the expected cycle time of carousel \( c \)
λc  the arrival rate to carousel c on-queue, \( \lambda_c = 1/E[CT_c] \)
ηc  the arrival rate to carousel c off-queue
νo  the arrival rate to picking station o on-queue
φo  the arrival rate to picking station o off-queue
f_qk  the flow rate from on-queue q to off-queue k

Variables:

\( \Delta_q \)  the flow rate on the conveyor that passes on-queue q in loads per time unit
\( SF^c_q \)  the stability factor for the conveyor segment that ends with on-queue q
\( SF^k \)  the stability factor for the conveyor segment that ends with off-queue k
\( SF_{sys} \)  the stability factor for the picking machine’s closed-loop conveyor

We can determine if a picking machine meets throughput by determining if each of the subsystems of the picking machine are stable at the required throughput: the conveyor, the picking station(s), and the carousel system(s). In a picking machine with a closed-loop conveyor, totes that are requested at a picking station may need to pass by other on- and off-conveyor queues prior to reaching their destination. For example, in Figure 1 a tote retrieved from storage carousel c destined for pick station 2 will have to pass by pick station 1’s on-queue. Such totes consume capacity on the conveyor. As developed in [4], the flow rate that passes on-queue i, \( \Delta_i \), is expressed as,

\[
\Delta_i = \sum_{q \in Q, q \neq i} \sum_{k \in K} f_{qk} \chi_{iqk} \quad \forall i \in Q. \tag{1}
\]

Stability conditions are calculated for carousel and pick station on-queues to determine if the conveyor system is stable. The stability factor from Bozer and Hsieh [4] is presented for carousel system c on-queue in our notation,

\[
SF^c = \frac{\lambda_c + \Delta_c}{V} \quad \forall c \in C. \tag{2}
\]

The rate of totes to picking station on-queues depends both on the order-fulfillment service rate at the pick station and the arrival rate of totes to the pick station. This is reflected in the stability factor for picking stations,

\[
SF^o = \frac{\min\{\mu_o, \phi_o\} + \Delta_o}{V} \quad \forall o \in O. \tag{3}
\]

To determine if a picking machine is stable, the service activities at pick stations and carousels are incorporated into our analysis. Off-queue stability functions for picking station are expressed as,

\[
SF^o = \frac{\phi_o}{\mu_o} \quad \forall o \in O. \tag{4}
\]

The service time at a carousel off-queue is equal to the expected cycle time and a carousel off-queue stability factor is shown below,

\[
SF^c_{off} = E[CT_c] \eta_c \quad \forall c \in C. \tag{5}
\]

For a picking machine to be stable, all three subsystems are required to be stable (i.e., the expected arrival rate must be less than the expected service rate). Therefore, a picking machine is stable if all on- and off-queue stability functions are less than one, which we present in our notation as,

\[
SF^c_q < 1, \quad \forall q \in Q. \tag{6}
\]

\[
SF^k < 1, \quad \forall k \in K. \tag{7}
\]

\[
SF_{sys} = \max\{SF^c_q, SF^k\} < 1. \tag{8}
\]

In the next section we evaluate the performance of our throughput stability-condition model for a picking machine.
4. Numerical Testing

We evaluate our throughput stability-condition model using a discrete-event simulation model of a picking machine over a wide range of operating scenarios. We first validate our model under common assumptions and then investigate the impact of accurately predicting stability when the following key assumptions of Bozer and Hsieh’s [4] model are violated: Poisson arrivals to the conveyor system and an infinite number of buffer spaces at loading stations.

In our testing we consider 36 picking machine designs by varying both the number of pick stations and the number of carousels between two and four. We normalize the pick station pick rate and the carousel system rate as a percentage of the average rate and vary the normalized rates among 0.75, 1.00, or 1.10. Totes are equally likely to be requested at each of the pick stations and pick rates for each pick station follow an independent Poisson process with a rate of 0.167 totes per second. The conveyor travels at a constant rate of 0.3333 windows per second and one window is assumed to accommodate one tote. All carousels are assumed to have the same expected cycle time of 11.50 seconds and tote retrieval requests are equally likely to be from each carousel system. We assume an average of 100 lines per tote; therefore, 1% of totes will be emptied at a pick station and removed from the system. The remaining 99% of totes are assumed to be returned to the storage system after use; hence, $SF_{off} = 0.99$ for each carousel off-queue.

Numerical results from our stability-condition model and simulation are provided in Table 1. The stability functions are equal for all carousel on-queues, as well as all picking station on-queues for the cases tested; hence, we only display the stability function value for one carousel and one picking station on-queue in the table of results. All simulation instances represent the average of five replications, each with 100,000 conveyor window movements and a warm-up period of 1,000 window movements.

To illustrate the numerical results of Table 1, consider the case with 2 pick stations, 2 carousels, and pick station and carousel normalized rates of 0.75. Therefore, $\lambda_c = 0.75/11.50 = 0.065$, $\mu_c = 0.167/0.75 = 0.222$, $\nu_s = \min\{0.222, 0.065\} = 0.065$, and $\phi_s = 0.75/11.50 = 0.065$. The flow rates to the carousel and pick station on-queue are calculated using (1), resulting in $\Delta_c = (2 - 1) \cdot 0.065 = 0.065$ and $\Delta_s = (2 - 1) \cdot 0.065 = 0.065$. The stability conditions for the carousel on-queue, pick station on-queue, and pick station off-queue are calculated using (2), (3) and (4), resulting in $SF_{on}^c = (0.065 + 0.065)/0.3333 = 0.39$, $SF_{on}^s = (0.065 + 0.065)/0.3333 = 0.39$ and $SF_{off}^c = 0.065/0.222 = 0.29$, respectively. The system stability condition is the maximum of $SF_{on}^c$, $SF_{on}^s$, and $SF_{off}^c$, which is 0.39. Because the system stability condition is less than 1.0, the analytical model deems the system stable. From our simulation, the average number of totes in the system is 15, which results in a stable system.

4.1 Validating our Model for Common Assumptions

First, we test our stability-condition model with exponential service times at both the carousel systems and the pick stations, as well as an infinite number of buffer spaces at all queues. The simulation results are displayed under the column entitled “EXPON and Infinite Buffers” in Table 1. For each instance tested, when the stability factor for the system is greater than or equal to 1.0, our simulated picking machine is unstable; when the stability factor for the system is less than 1.0, the picking machine is stable. Therefore, the stability-condition model correctly predicts stability with exponential service times and infinite buffer spaces. Next, we determine if our stability model can accurately predict stability when these assumptions are violated.

4.2 Validating our Model for Non-Poisson Arrivals

To evaluate the impact of non-Poisson arrivals to the conveyor, we violate the assumption of exponential service times of the carousel systems and instead simulate a horizontal carousel system with an S/R machine performing batch retrievals [14]. We denote the empirical distribution of a carousel cycle time as $CT$. Simulation results are provided in Table 1 under the column “CT and Infinite Buffers.” In all instances tested, our stability-condition model provided the same results as our simulation model. Therefore, the stability-condition model accurately predicts whether the picking machine will be stable or not regardless of the arrival rate distribution of totes from the carousels to the conveyor. The carousel cycle time, with a coefficient of variation around 0.27, has lower variability than an exponential distribution [14]; therefore, the picking machine throughput with exponential cycle times is less than the picking machine throughput with a carousel cycle-time arrival distribution. Additionally, due to the increased variability, the average number in the system is greater for the exponential case than for the carousel cycle-time distribution.

4.3 Validating our Model for a Finite Number of Buffer Spaces

With an infinite number of buffer spaces at queues, the operations of the pick stations and the carousels are never blocked regardless of the number of totes waiting to be loaded onto the conveyor. However, with a finite number of buffer spaces, blocking can occur causing carousels and pick stations to be idle, resulting in a loss of throughput from
### Table 1: Results Validating our Throughput Stability Model Against Simulation

<table>
<thead>
<tr>
<th>#</th>
<th>PS #</th>
<th>Picking Station On</th>
<th>Picking Station Off</th>
<th>System</th>
<th>EXPON and Infinite Buffers</th>
<th>CT and Infinite Buffers</th>
<th>CT and 5 Buffer Loc.</th>
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<td>#</td>
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<td>Car. Norm. Rate</td>
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those subsystems. The impact of a finite number of buffer spaces on picking machine throughput is tested through simulation. We continue to assume Poisson arrivals to the conveyor system and an infinite number of buffer spaces at off-queues, but now vary the number of buffer spaces at both carousel and pick station on-queues.

Table 2 displays the percent loss of throughput over a picking machine with an infinite number of buffer spaces. Whereas the throughput of the picking machine decreases as the number of buffer spaces decreases, the loss of throughput is on average less than 1.5% when five buffer spaces are provided at each queue. Consequently, we assume that five buffer spaces are adequate to achieve a given throughput performance and use this value in the next section.

### 4.4 Validating our Model for Non-Poisson Arrivals and a Finite Number of Buffer Spaces

Finally, we analyze the impact of both non-Poisson arrival rates and a finite number of buffer spaces on a picking machine’s throughput. We set the number of buffer spaces at each queue to five and use an empirical carousel cycle-time distribution as the arrival rate distribution to the conveyor from the carousel. The results of the simulation are displayed in the column “CT and 5 Buffer Spaces” of Table 1. The maximum loss of throughput over the case with CT and infinite capacity is 3.42%, and on average is 0.37%. Therefore, our stability-condition model is sufficient in determining stability of the conveyor for the case of non-Poisson arrivals and a finite number of buffer spaces.

### 5. Conclusions

By extending Bozer and Hsieh’s model [4], we developed a throughput stability-condition model based on queuing
theory to determine if a picking machine’s conveyor system will be able to meet a throughput requirement. Our
model was validated using a discrete-event simulation model. As illustrated by testing, our stability-condition model
accurately predicted stability even when assumptions about Poisson arrival rates to the conveyor and infinite buffer
spaces at on-queues are violated. In addition, we showed that five buffer spaces are adequate to achieve a given
throughput performance. The intuition behind our results is that the stability condition depends only on the first
moments. An open research issue is the generalization of our results for non-stationary rates from the pick stations
and carousel systems.

A picking machine can be divided into three subsystems: a storage subsystem, a pick station subsystem, and a
closed-loop conveyor subsystem. Our work here is capable of analyzing the closed-loop conveyor subsystem. Analysis
of the remaining two subsystems would need to consider the inventory differences between a picking machine and a
competing stock-to-picker system, as well as a model to determine the number of pick stations required to meet
throughout. These issues, as well as a throughput model of a carousel storage subsystem, are presented in [14].

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